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Angular Momentum Oscillations in Rotating Two-Dimensional Fermi Gases Confined by Harmonic Traps

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Abstract: We investigate the angular momentum of a two-dimensional non-interacting Fermi gas confined in a rotating harmonic trap. By employing a quantum statistical approach, analytical expressions for the angular momentum are derived and analyzed as functions of particle number and rotation frequency. The rotational motion introduces a shell structure in the energy spectrum, giving rise to oscillatory behavior analogous to the de Haas–van Alphen oscillations observed in electronic systems subjected to magnetic fields. Numerical calculations reveal that these oscillations are most pronounced for weak rotation and small particle numbers, while they become progressively damped as the particle number increases. The results emphasize the close analogy between ultracold rotating gases and charged particles in magnetic fields and provide further insight into quantum rotational effects in confined Fermi systems.

Keywords: Ultracold Fermi gases, Angular momentum, Quantum oscillations, Rotating traps, Harmonic confinement.

1. Introduction

The study of ultracold quantum gases has become one of the most active research areas in modern atomic physics. These systems provide highly controllable platforms for investigating a wide range of quantum many-body phenomena that are often difficult to access in conventional condensed-matter systems. Since the first experimental realization of Bose–Einstein condensation in dilute atomic gases, ultracold atoms have emerged as powerful quantum simulators capable of reproducing the behavior of strongly correlated electronic systems, superfluids, and quantum Hall states [1,2,3].

Among ultracold quantum systems, degenerate Fermi gases have attracted considerable attention because of their close analogy with electrons in solids. Their properties can be precisely controlled through external trapping potentials, rotation, artificial gauge fields, and interaction tuning techniques. Such flexibility has enabled the exploration of fundamental quantum phenomena including superfluidity, collective excitations, vortex formation, and quantum oscillations [5].

Rotation plays a particularly important role in trapped quantum gases. In a rotating reference frame, the Hamiltonian acquires an additional term proportional to the angular momentum operator, making the dynamics of neutral atoms analogous to those of charged particles subjected to a magnetic field. This analogy has

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stimulated extensive theoretical and experimental investigations aimed at reproducing magnetic-field-induced phenomena in ultracold atomic systems [6,7,8].

One of the most remarkable manifestations of Landau quantization in condensed matter physics is the appearance of quantum oscillations, such as the de Haas–van Alphen effect, in which the magnetization oscillates as a function of magnetic field or chemical potential. Similar oscillatory phenomena have been predicted and observed in rotating ultracold Fermi gases, where the angular momentum plays a role analogous to magnetization. These oscillations originate from the discrete structure of the energy spectrum and provide valuable information about the quantum states occupied by the particles.

The angular momentum is a fundamental observable in rotating quantum systems because it characterizes the response of the gas to external rotation and reflects the underlying shell structure of the energy spectrum. Unlike classical gases, where the angular momentum is directly related to the moment of inertia, quantum gases exhibit nontrivial deviations from classical behavior due to the quantization of energy levels. As a consequence, the angular momentum may display oscillatory behavior as system parameters such as particle number, chemical potential, or rotation frequency are varied [9,10,11,12,13].

In the present work, we investigate the angular momentum of a two-dimensional non-interacting Fermi gas confined in a rotating isotropic harmonic trap. Analytical expressions are derived for the angular momentum, and its behavior is analyzed as a function of the particle number and the rotation frequency. Particular attention is devoted to the emergence of quantum oscillations and their relation to shell-filling effects. The obtained results highlight the strong analogy between rotating ultracold atomic gases and electronic systems subjected to magnetic fields, providing further insight into quantum rotational phenomena in confined Fermi systems.

2. Theoretical Description of the Rotating Fermi Gas

The system under consideration in this study is made up of N fermionic particles with no electric charge (neutral atoms) of mass m^* that can only travel in a plane, i.e., (x, y).

This restriction is due to a strong confinement on the (OZ) axis such that one can consider the movement along this axis as frozen. We add to the trapping potential, which maintains the particles in a two-dimensional region, another potential which makes the system rotate with an angular velocity $\vec{\Omega} = \Omega\vec{e_z}$, The Hamiltonian governing the evolution of this system is given by [14]

$$H = \frac{1}{2m^*}\mathbf{P}^2 + \frac{1}{2}m^*\boldsymbol{r}^2 - \boldsymbol{\Omega}.\mathbf{L} \tag{1}$$

The momentum and angular momentum operators of the particle are denoted by $\boldsymbol{P}$ and $\mathbf{L}$, respectively. It is useful to introduce, in order to explain the impact of rotation on the attributes of the system. In the solid-state environment, the Hamiltonian (1) is an analog of the Hamiltonian describing an electron travelling in a magnetic field The angular momentum is analogous to magnetization, and the angular frequency is analogous to the magnetic field. It should be noted, however, that in the solid-state situation, there is usually no external trapping potential.

The Hamiltonian (1) gives rise to the following spectrum [15,16]:

$$E_{nm} = \left(n + \frac{1}{2}\right)\hbar\Omega_+ + \left(m + \frac{1}{2}\right)\hbar\Omega_- \quad ; n, m = 0,1,2, \dots. \tag{2}$$

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The mean value of the total angular momentum of a system of particle seach of mass is [17]

$$\mathbf{L} = m^* \int (\mathbf{r} \times \mathbf{J}) d\mathbf{r} \tag{3}$$

Knowledge of particle movement and atomic density is also required for a better calculation of angular momentum

The current density in the rotating frame is:

$$\mathbf{J} = \mathbf{J}_{kin} - (\boldsymbol{\Omega} \times \mathbf{r})\rho(\mathbf{r}) = \mathbf{J}_{kin} - \Omega r \rho(\mathbf{r}) \mathbf{e}_\varphi \tag{4}$$

where $\mathbf{J}_{kin}$ The kinematical part of the current density, $\rho(\mathbf{r})$ the particle density

For a rotating isotropic harmonic trap, this current density is given by [18]:

$$\mathbf{J}(\mathbf{r}) = \mathbf{e}_\varphi \frac{r}{2\pi a^2} \mathcal{L}_\mu^{-1} \left\{ \frac{1}{\xi} \left(\frac{\omega_0 \sinh(\xi\hbar\Omega)}{\sinh^2(\xi\hbar\omega_0)} - \frac{\Omega}{\sinh(\xi\hbar\omega_0)} \right) \exp\left[-\frac{\mathbf{r}^2}{a^2} \left(\coth(\xi\hbar\omega_0) - \frac{\cosh(\xi\hbar\Omega)}{\sinh(\xi\hbar\omega_0)} \right) \right] \right\} \tag{5}$$

That can be rewritten as

$$\mathbf{J}(\mathbf{r}) = \mathbf{e}_\varphi \frac{r}{2\pi a^2} \mathcal{L}_\mu^{-1} \left\{ \frac{1}{\xi} \lambda \exp[-\beta \mathbf{r}^2] \right\} \tag{6}$$

Where

$$\begin{cases} \lambda = \dfrac{\omega_0 \sinh(\xi\hbar\Omega)}{\sinh^2(\xi\hbar\omega_0)} - \dfrac{\Omega}{\sinh(\xi\hbar\omega_0)} = \dfrac{\omega_0}{\sinh(\xi\hbar\omega_0)} \left(\dfrac{\sinh(\xi\hbar\Omega)}{\sinh(\xi\hbar\omega_0)} - \alpha \right) \\ \beta = \dfrac{\cosh(\xi\hbar\omega_0) - \cosh(\xi\hbar\Omega)}{a^2 \sinh(\xi\hbar\omega_0)} = \dfrac{2\sinh(\xi\hbar\Omega_+/2)\sinh(\xi\hbar\Omega_-/2)}{a^2 \sinh(\xi\hbar\omega_0)} \end{cases} \tag{7}$$

The parameter $\alpha = \Omega/\omega_0$ is the rotation ratio and $\Omega_\pm = \omega_0 \pm \Omega$. The vector product in (1) grives after (4)

$$\mathbf{r} \times \mathbf{J} = \mathbf{e}_z \frac{r^2}{2\pi a^2} \mathcal{L}_\mu^{-1} \left\{ \frac{1}{\xi} \lambda \exp[-\beta r^2] \right\} \tag{8}$$

The total angular momentum given by (1) becomes

$$\mathbf{L} = \mathbf{e}_z \frac{m^*}{2\pi a^2} \mathcal{L}_\mu^{-1} \left\{ \frac{1}{\xi} \lambda \int r^2 \exp[-\beta r^2] d\mathbf{r} \right\} = \mathbf{e}_z \frac{m^*}{a^2} \mathcal{L}_\mu^{-1} \left\{ \frac{1}{\xi} \lambda \overbrace{\int_0^\infty r^3 \exp[-\beta r^2] dr}^{I} \right\} \tag{9}$$

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the value of the integral I is

$$I = \int_0^\infty r^3 \exp[-\beta r^2] dr = \frac{1}{2\beta^2} \tag{10}$$

If we substitute this result in (7) we obtain

$$\mathbf{L} = \mathbf{e}_z \frac{m^*}{2a^2} \mathcal{L}_\mu^{-1} \left\{ \frac{1}{\xi} \frac{\lambda}{\beta^2} \right\} \tag{11}$$

Upon using (5) we find

$$\frac{\lambda}{\beta^2} = \frac{\omega_0 a^4}{4} \left(\overbrace{\frac{\sinh(\xi\hbar\Omega)}{\sinh^2(\xi\hbar\Omega_+/2)\sinh^2(\xi\hbar\Omega_-/2)}}^{T_1} - \alpha \overbrace{\frac{\sinh(\xi\hbar\omega_0)}{\sinh^2(\xi\hbar\Omega_+/2)\sinh^2(\xi\hbar\Omega_-/2)}}^{T_2} \right) \tag{12}$$

If we recall that

$$\begin{cases} \Omega = \dfrac{\Omega_+ - \Omega_-}{2} \\ \omega_0 = \dfrac{\Omega_+ + \Omega_-}{2} \end{cases} \Rightarrow \begin{cases} \sinh(\xi\hbar\Omega) = \sinh\left(\dfrac{\xi\hbar\Omega_+}{2}\right)\cosh\left(\dfrac{\xi\hbar\Omega_-}{2}\right) - \cosh\left(\dfrac{\xi\hbar\Omega_+}{2}\right)\sinh\left(\dfrac{\xi\hbar\Omega_-}{2}\right) \\ \sinh(\xi\hbar\omega_0) = \sinh\left(\dfrac{\xi\hbar\Omega_+}{2}\right)\cosh\left(\dfrac{\xi\hbar\Omega_-}{2}\right) + \cosh\left(\dfrac{\xi\hbar\Omega_+}{2}\right)\sinh\left(\dfrac{\xi\hbar\Omega_-}{2}\right) \end{cases} \tag{13}$$

We should have the following

$$T_1 = \frac{\cosh\left(\frac{\xi\hbar\Omega_-}{2}\right)}{\sinh\left(\frac{\xi\hbar\Omega_+}{2}\right)\sinh^2\left(\frac{\xi\hbar\Omega_-}{2}\right)} - \frac{\cosh\left(\frac{\xi\hbar\Omega_+}{2}\right)}{\sinh^2\left(\frac{\xi\hbar\Omega_+}{2}\right)\sinh\left(\frac{\xi\hbar\Omega_-}{2}\right)} \tag{14}$$

Now, if we use the following expansions

$$\begin{cases} \sinh^{-1}(x) = 2\displaystyle\sum_{n=0}^{\infty} e^{-(2n+1)x} \\ \dfrac{\cosh(x)}{\sinh^2(x)} = 2\displaystyle\sum_{m=0}^{\infty} (2m+1)e^{-(2m+1)x} \end{cases} \tag{15}$$

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We find for the first and second terms in the second member of(13) the following results respectively

$$\frac{\cosh\left(\frac{\xi\hbar\Omega_-}{2}\right)}{\sinh\left(\frac{\xi\hbar\Omega_+}{2}\right)\sinh^2\left(\frac{\xi\hbar\Omega_-}{2}\right)} = \left(2\sum_{n=0}^{\infty} e^{-\left(n+\frac{1}{2}\right)\xi\hbar\Omega_+}\right)\left(2\sum_{m=0}^{\infty}(2m+1)e^{-\left(m+\frac{1}{2}\right)\xi\hbar\Omega_-}\right)$$
$$= 4\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(2m+1)e^{-\left(n+\frac{1}{2}\right)\xi\hbar\Omega_+-\left(m+\frac{1}{2}\right)\xi\hbar\Omega_-} \tag{16}$$

And

$$\frac{\cosh\left(\frac{\xi\hbar\Omega_+}{2}\right)}{\sinh^2\left(\frac{\xi\hbar\Omega_+}{2}\right)\sinh\left(\frac{\xi\hbar\Omega_-}{2}\right)} = \left(2\sum_{n=0}^{\infty}(2n+1)e^{-\left(n+\frac{1}{2}\right)\xi\hbar\Omega_+}\right)\left(2\sum_{m=0}^{\infty} e^{-\left(m+\frac{1}{2}\right)\xi\hbar\Omega_-}\right)$$
$$= 4\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(2n+1)e^{-\left(n+\frac{1}{2}\right)\xi\hbar\Omega_+-\left(m+\frac{1}{2}\right)\xi\hbar\Omega_-} \tag{17}$$

Thus, we write for (12)

$$T_1 = 8\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(m-n)e^{-\left(n+\frac{1}{2}\right)\xi\hbar\Omega_+-\left(m+\frac{1}{2}\right)\xi\hbar\Omega_-} \tag{18}$$

Following the same method as above, we have

$$T_2 = \frac{\cosh\left(\frac{\xi\hbar\Omega_-}{2}\right)}{\sinh\left(\frac{\xi\hbar\Omega_+}{2}\right)\sinh^2\left(\frac{\xi\hbar\Omega_-}{2}\right)} + \frac{\cosh\left(\frac{\xi\hbar\Omega_+}{2}\right)}{\sinh^2\left(\frac{\xi\hbar\Omega_+}{2}\right)\sinh\left(\frac{\xi\hbar\Omega_-}{2}\right)} \tag{19}$$

Using (14) and (15) we find

$$T_2 = 8\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(n+m+1)e^{-\left(n+\frac{1}{2}\right)\xi\hbar\Omega_+-\left(m+\frac{1}{2}\right)\xi\hbar\Omega_-} \tag{20}$$

Inserting (16) and (18) into (10) we obtain

$$\frac{\lambda}{\beta^2} = 2\omega_0 a^4\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}[(m-n)-\alpha(n+m+1)]e^{-\xi E_{nm}} \tag{21}$$

We substitute now the result (19) in (9) to find that:

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$$\mathbf{L} = \mathbf{e}_z m^* \omega_0 a^2 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} [(m-n) - \alpha(n+m+1)]\, \mathcal{L}_\mu^{-1} \left\{ \frac{e^{-\xi E_{nm}}}{\xi} \right\} \tag{22}$$

Recalling that $a^2 = \hbar / m^* \omega_0$, we can rewrite this expression as

$$\mathbf{L} = \mathbf{e}_z m^* \omega_0 a^2 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} [(m-n) - \alpha(n+m+1)]\, \mathcal{L}_\mu^{-1} \left\{ \frac{e^{-\xi E_{nm}}}{\xi} \right\} \tag{23}$$

Finally, if we use the identity

$$\Theta(\mu - E_{nm}) = \mathcal{L}_\mu^{-1} \left\{ \frac{\exp(-\xi E_{nm})}{\xi} \right\} \tag{24}$$

The expression of the mean value of the total angular momentum is

$$\mathbf{L} = \hbar \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \left\{ (m-n) - \frac{\Omega}{\omega_0}(n+m+1) \right\} \Theta(\mu - E_{nm})\, \mathbf{e}_z \tag{25}$$

It consists of two parts, classical and quantitative

$$\mathrm{R} = \frac{\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} (m-n) \Theta(\mu - E_{nm})}{\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \{\alpha(n+m+1)\} \Theta(\mu - E_{nm})} = \frac{L_Q}{L_{cl}} \tag{26}$$

3. Results and Discussion

The behavior of the ratio R as a function of the particle number N is illustrated in Figures 1 and 2 for different values of the rotation parameter α. This ratio characterizes the deviation of the quantum angular momentum from its classical counterpart and therefore provides a useful measure of quantum rotational effects in the confined Fermi gas.

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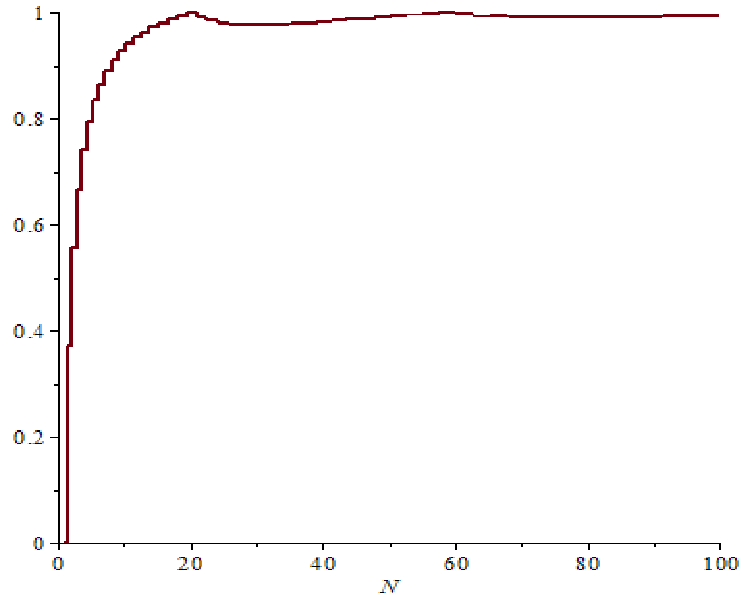

Figure1: The ratio R, for $\alpha = 0.9$,and $T = 0$. The oscillatory contribution is insignificant at high particle numbers.

Figure 1 shows the evolution of R for a rotating two-dimensional Fermi gas. A rapid increase of the ratio is observed for small particle numbers, followed by a gradual saturation toward unity. The approach to R indicates that the quantum angular momentum converges to its classical value as the number of occupied states increases. This behavior reflects the transition from a strongly quantum regime dominated by finite-size effects to a regime where the collective behavior of many particles becomes increasingly classical.

A notable feature of Figure 1 is the presence of small oscillations around the classical limit. These oscillations arise from the discrete nature of the single-particle energy spectrum and are associated with shell-filling effects. As successive energy levels become occupied, the angular momentum changes non-uniformly, leading to fluctuations in the ratio R. Such oscillatory behavior is analogous to the de Haas–van Alphen oscillations observed in electronic systems subjected to magnetic fields.

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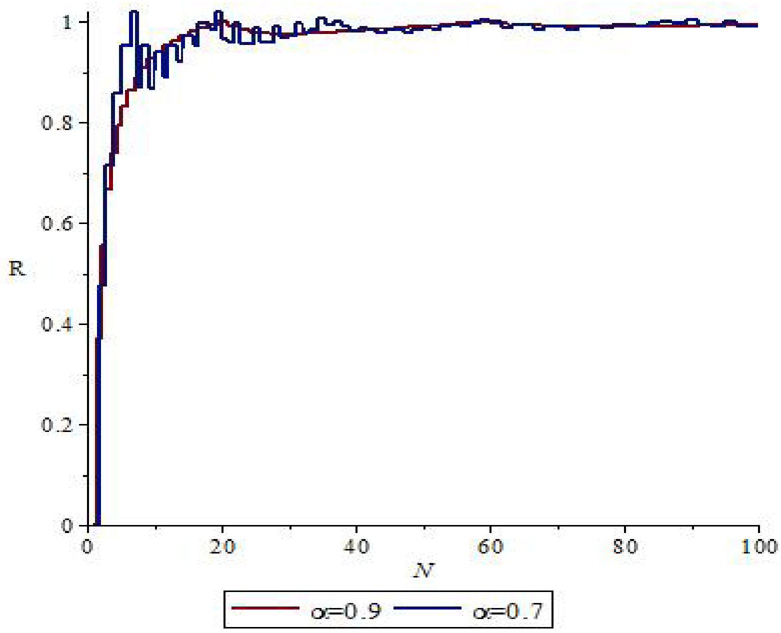

Figure2:The ratio R, for $\alpha = 0.9$,and $\alpha = 0.7$ at $T = 0$, . The oscillatory contribution is insignificant at high particle numbers.

Figure 2 compares the behavior of R for two different rotation parameters, namely $\alpha = 0.9$ and $\alpha = 0.7$. Although both curves exhibit the same general trend, significant differences appear in the oscillation amplitude. For $\alpha = 0.7$, the oscillations are more pronounced and remain visible over a larger range of particle numbers. In contrast, for $\alpha = 0.9$, the oscillations are strongly suppressed and the ratio approaches unity more smoothly.

The dependence of the oscillatory behavior on the rotation parameter can be understood from the structure of the energy spectrum. A decrease in the rotation parameter enhances the role of discrete energy levels and increases the sensitivity of the system to shell closures. Consequently, stronger quantum oscillations are observed. On the other hand, larger values of α lead to a denser effective spectrum, reducing the contribution of individual shell fillings and therefore damping the oscillatory behavior.

The strongest oscillations occur in the low-particle-number region $N < 20$, where the occupation of each additional quantum state significantly affects the total angular momentum. As the particle number increases, many states contribute simultaneously to the thermodynamic properties of the gas, resulting in a progressive smoothing of the oscillations. For $N > 40$, the curves become nearly flat and remain very close to unity, indicating that finite-size quantum effects are largely suppressed.

The results clearly demonstrate that both the particle number and the rotation parameter play crucial roles in determining the visibility of quantum oscillations. Small particle numbers combined with weak rotation provide the most favorable conditions for observing shell effects and quantum fluctuations of the angular momentum. Conversely, large particle numbers and stronger rotation drive the system toward the classical regime, where the angular momentum is accurately described by classical predictions.

Overall, the behavior displayed in Figures 1 and 2 confirms the existence of quantum oscillations in the angular momentum of rotating confined Fermi gases. These oscillations originate from the quantized structure of the energy spectrum and constitute a direct manifestation of quantum finite-size effects. The observed analogy with the de Haas–van Alphen effect further highlights the close correspondence between rotating neutral atomic gases and charged particles in magnetic fields.

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4. Conclusion:

In this work, we have investigated the angular momentum of a two-dimensional non-interacting Fermi gas confined in a rotating isotropic harmonic trap. Using a quantum statistical description, analytical expressions for the angular momentum were derived and employed to study its dependence on the particle number and the rotation parameter.

Our results reveal the existence of pronounced quantum oscillations in the angular momentum, which originate from the discrete shell structure of the single-particle energy spectrum. These oscillations are particularly significant for small particle numbers, where finite-size quantum effects strongly influence the rotational properties of the system. As the particle number increases, the oscillation amplitude gradually decreases and the angular momentum approaches its classical limit, reflecting the transition from quantum to quasi-classical behavior.

The influence of the rotation parameter was also examined. We found that weaker rotation enhances the visibility of shell effects and leads to larger oscillation amplitudes, whereas stronger rotation tends to smooth out the oscillatory behavior. This demonstrates that both the particle number and the rotation strength play a crucial role in determining the magnitude of quantum rotational effects.

Furthermore, the observed oscillatory behavior exhibits a close analogy with the de Haas–van Alphen effect in electronic systems subjected to magnetic fields. In this correspondence, the angular momentum of the rotating gas plays a role analogous to the magnetization of an electron gas, highlighting the deep connection between rotating neutral atomic systems and charged particles in magnetic environments.

The present study contributes to the understanding of quantum rotational phenomena in ultracold Fermi gases and provides further insight into finite-size effects in confined quantum systems. Future investigations may include the effects of finite temperature, interparticle interactions, anisotropic confinement, and spin–orbit coupling, which could lead to richer dynamical and thermodynamic behaviors.

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